DESIGN OPTIMIZATION OF REINFORCED ORDINARY AND HIGH STRENGTH CONCRETE BEAMS

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Résumé : Cet article présente une approche pour minimiser séparément le coût et le poids du béton armé ordinaire et le béton à haute résistance des poutres en T aux états limites selon l'Eurocode2 (EC-2). La première fonction objectif comprend le coût de béton, de l'acier et du coffrage et la deuxième fonction objectif représente le poids de la poutre en T. Toutes les fonctions de contraintes sont définies pour répondre aux exigences de conception de l'Eurocode2 et des règles pratiques en vigueur. Le processus d'optimisation est développé grâce à l'utilisation de l'algorithme du Gradient Réduit Généralisée. Deux exemples sont traités dans le but d'illustrer l'applicabilité du modèle de conception proposée et la méthodologie développée. Il est conclu que cette approche est économiquement plus efficace comparativement aux méthodes de conception classiques utilisées par les concepteurs et les ingénieurs et peut être facilement étendue pour d'autres sections sans altération majeure.

Mots-clés: minimisation du coût et du poids, poutres en béton armé ordinaire et à haute résistance, Eurocode2 (EC-2), optimisation non-linéaire, algorithme.

Abstract. This paper presents a method for minimizing separately the cost and weight of reinforced ordinary and High Strength Concrete (HSC) T-beams at limit state according to Eurocode2 (EC-2). The first objective function includes the costs of concrete, steel and formwork and the second objective function deals with the weight of the T-beam. All the constraints functions are set to meet design requirements of Eurocode2 and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. Two example problems are considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods used by designers and engineers and can be extended to deal with other sections without major alterations.

Keywords: Cost and weight minimization, Reinforced ordinary and high strength concrete beams, Eurocode2 (EC-2), Non-linear optimization, Algorithm.

1. Introduction

Structural elements with T-shaped sections are frequently used in industrial construction. They are used for repeated and large structures because they are cost effective when using optimum cost design model which is of great value for designers and engineers. Compression reinforcement is not often required when designing the T-beams sections. One of the great advantages of T-beams sections is the economy in the amount of steel needed for reinforcement. The objective function is usually simplified to represent the weight, disregarding the costs of shaping and the construction details. However, the economy aspects in terms of costs and gain achieved should be the area where scope exists for extending the research works [1, 2, 3, 4].

Recent developments in the technology of materials have led to the use of the high strength concrete; this is mainly due to its efficiency and economy. The reduction in the quantities of construction materials has enabled both a gain in weight reduction and in foundation's cost. HSC has a high compressive strength in the range of 55 to 90MPa; not only has it the advantage of reducing member size and story height but also the volume of concrete and the area of formwork. In terms of the amount of steel reinforcement, there is a substantial difference between the normal strength concrete structures compared to high strength concrete structures [5,6]. In this work, not only does it presents the minimum weight design but it presents a detailed objective function that considers the ratios cost not the absolute cost with sensitivity analysis of this cost ratios as well. It considers both shaping and material costs. The generalized reduced gradient (GRG) method is used to solve nonlinear programming problems. It is a very reliable and robust algorithm; also, various numerical methods have been used in engineering optimization [7, 8, 9, 10, 11, 12].

the second objective function represents the weight of the T-beam, all the constraints functions are set to meet the ultimate strength and serviceability requirements of Eurocode2 and current practices rules. The optimization process is developed through the use of the Generalized Reduced Gradient algorithm. Two example problems are considered in order to illustrate the applicability of the proposed design model and solution methodology. It is concluded that this approach is economically more effective comparing to conventional design methods applied by designers and engineers and can be extended to deal with other sections without major alterations.

2. Limit state design of reinforced concrete T-section under bending

In accordance with the Eurocode 2 EC-2 [13], the assumptions used at limit state for the typical reinforced T-beam cross section are respectively illustrated in Fig. 1(a), (b), (c).

In the linear strain diagram of Fig.1.b, the

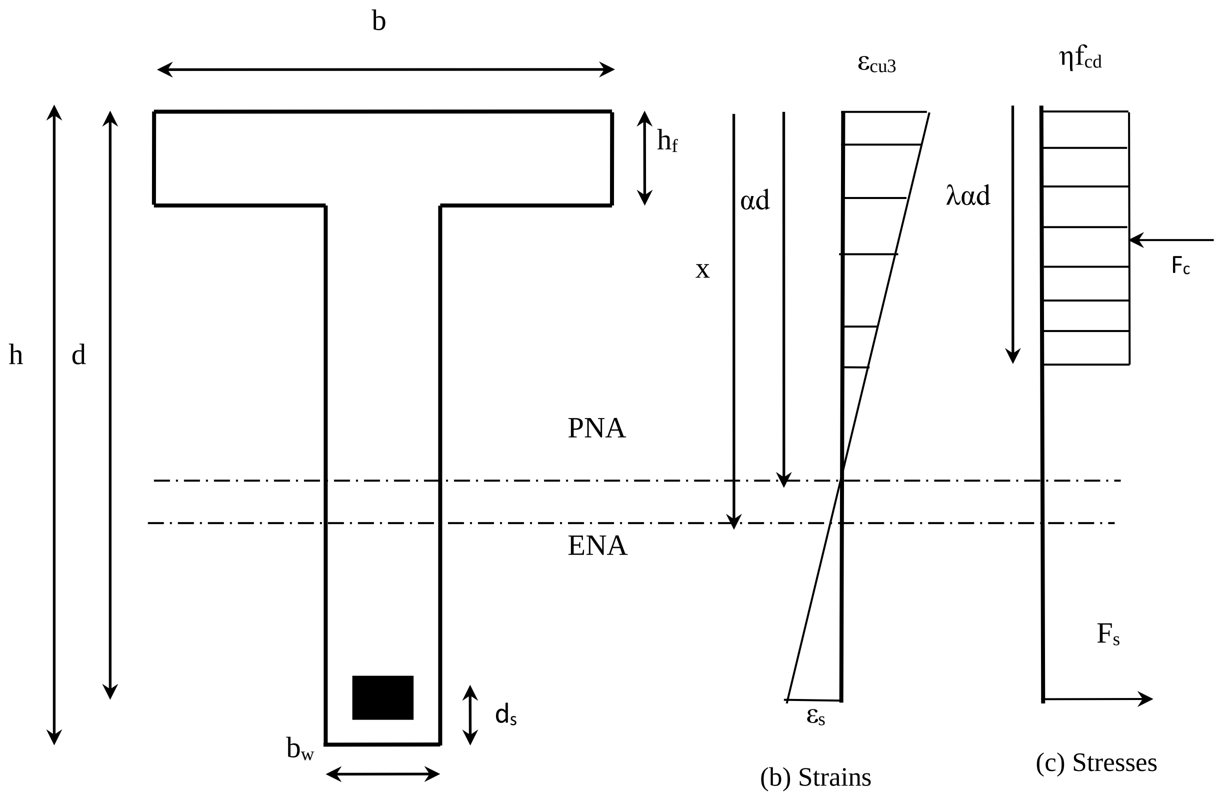

Figure 1 (a) Typical T-beam cross section; (b) strains at ultimate limit state and (c) stresses at ultimate limit state.

symbols ε_s and ε_{cu3} designate steel strain and ultimate strain for the rectangular stress distribution compressive concrete design stress-strain relation. The parameter α represents the relative depth of compressive concrete zone and the neutral plastic axis is located at the distance α d from the upper fiber for the ultimate limit state design and x is the depth of elastic neutral axis for serviceability limit state design. In the assumed uniformly distributed stress diagram of Figure 1.c, f_{cd} is the design value of concrete compressive strength, γ_c the partial safety factor for concrete and f_{ck} is characteristic compressive cylinder strength of ordinary or HSC at 28 days. In accordance with EC-2, the possibility is offered to work with a rectangular stress distribution. This requires the introduction of a factor λ for the depth of the compression zone and a factor η for the design strength. The λ and η factors are both linearly dependent on the characteristic strength f_{ck} in accordance with the following equations:

$$\lambda = 0.8 - \frac{f_{ck}-50}{400} \quad (1)$$

$$\mu = 1.0 - \frac{f_{ck}-50}{200} \quad (2)$$

With: $50 \leq f_{ck} \leq 90$MPa and λ=0.8,η =1.0 for $f_{ck} \leq 50$MPa

F_c and F_s denote the resultants of internal forces in HSC section and reinforcing steel respectively.

The design yield strength of steel reinforcement $f_{yd}=f_{yk}/\gamma_s$ where, f_{yk} is the characteristic elastic limit of steel and γ_s is the partial safety factor. In addition, the steel strain is considered unlimited in accordance with the Eurocode2 provisions. In this paper, for an optimal use of steel, the strain must always be greater or equal to elastic limit strain $\varepsilon_{yd}=f_{yd}/E_s$ where E_s represents the elasticity modulus for steel.

3. Formulation of the optimization of reinforced concrete T-beams in flexure

3.1 Design variables

The design variables selected for the optimization are presented in Table 1.

Table 1: Definition of design variables

Design variables	Defined variables
b	Effective width of compressive flange
b_w	Web width
h	Total depth
d	Effective depth
h_f	Flange depth
A_s	Area of tension reinforcement
α	Relative depth of compressive concrete zone

3.2 Objective functions

3.2.1 Cost function

The objective function to be minimized in the optimization problems the total cost of construction material per unit length of the beam. This function can be defined as:

$$C_0 = C_c(b_w h + (b - b_w)h_f) + C_s A_s$$
$$+C_f[b + 2h] \rightarrow \text{Minimum} \quad (3)$$

$$C_0 = C_c LC \quad (4)$$

Thus, the cost function to be minimized can be written as follows:

$$C = b_w h + (b - b_w)h_f + \left(\frac{C_s}{C_c}\right) A_s$$
$$+\left(\frac{C_f}{C_c}\right)[b + 2h] \rightarrow \text{Minimum} \quad (5)$$

The values of the cost ratios C_s/C_c and C_f/C_c vary from one country to another and may eventually vary from one region to another for certain countries [14,15].

3.2.2 Weight function

The weight function to be minimized can be written as follows:

$$W = (b_w h + (b - b_w)h_f)\rho \longrightarrow \text{Minimum} \quad (6)$$

Where:

ρ is the density of the reinforced concrete T-beams and W is the unit weight per unit length of the reinforced concrete T- beams.

3.3 Design constraints

a)- Behavior constraints:

$$M_{Ed} \leq \eta f_{cd}(b - b_w)h_f(d - 0{,}50h_f) + \eta\lambda f_{cd} b_w d^2 \alpha(1 - 0{,}5\lambda\alpha) \quad (7)$$

(External moment ≤Resisting moment of the cross section)

$$\alpha = \left(\frac{f_{yd}}{f_{cd}}\right)\left(\frac{A_s}{\eta\lambda b_w d}\right) - \frac{(b-b_w)h_f}{\lambda b_w d} \quad (8)$$

(Internal force equilibrium)

$$\frac{A_s}{b_w d} \geq p_{min} \quad (9)$$

(Minimum steelpercentage)

$$\frac{As}{b_w\, h + (b - b_w)h_f} \leq p_{max} \quad (10)$$

(Maximum steel percentage)

In equations (2) and (3) above, it is assumed that the neutral axis position is under the beam flange which ensures that the section is behaving as the T-beam section shown in Fig. 1.a.

Conditions on strain compatibility in steel:

$$\varepsilon_{cu3}\left(\left(\frac{1}{\alpha}\right) - 1\right) \geq \frac{f_{yd}}{E_s} \quad (11)$$

(In the case of Pivot B, optimal use of steel requires that strains in steelmust be limited to plastic region at the ULS)

$$\lambda\alpha(1 - 0{,}5\lambda\alpha) \leq \mu_{limit} \quad (12)$$

(Compression reinforcement is not required)

b)- Shear strength constraint:

$$V_{Ed} \leq V_{Rd,max} = \nu_1 \frac{f_{cd} b_w z}{tg(\theta) + cotg(\theta)} \quad (13)$$

(External shear force ≤Resisting shear force)

c)- Deflection constraint :

$$\frac{5wL^4}{384\, E_{cm} I_c} \leq \delta_{lim} \quad (14)$$

$$I_c = \frac{b_w h^3}{3} + \frac{(b-b_w)h^3}{3} + nA_s d^2 - A_h x^2 \quad (15)$$

$$A_h = b_w h + (b - b_w)h_f + nA_s \quad (16)$$

$$x = \frac{\frac{b_w h^2}{2} + \frac{(b-b_w)h_f^2}{2} + nA_s d}{A_h} \quad (17)$$

d)- Geometric design variables constraints including rules of current practice:

$$h \geq \frac{L}{16} \quad (18)$$

$$\frac{d}{h} = 0.90 \quad (19)$$

$$0.20 \leq \frac{b_w}{d} \leq 0.50 \quad (20)$$

$$\frac{(b - b_w)}{2} \leq \frac{L}{10} \quad (21)$$

$$\frac{b}{h_f} \leq 8 \quad (22)$$

$$h_f \geq h_{fmin} \quad (23)$$

$$\frac{b}{b_w} \geq 3 \quad (24)$$

e)- Non-negativity variables:

$$b,\ b_w, h, d, h_f, A_s, \alpha \geq 0 \quad (25)$$

Where:
μ_{limit} is the limit value of reduced moment
θ is the angle between concrete compression struts and the main chord.
ν_1 is a non dimensional coefficient; ν_1=0.60(1-f_{ck}/250)
z is the lever arm, z=0.9d
h_{fmin} is the minimum depth of flange

3.4. Formulation of minimum cost design problem of reinforced concrete T-beams

Thus the formulation of the optimum cost design of reinforced concrete T-beams under limits state can be mathematically stated as follows:

Given the characteristics of material, loading data and constant parameters.

Find the design variables b, b_w, h, d, h_f, A_s, and α that minimize total cost of construction material per unit length of T-beam such that:

$$C = b_w h + (b - b_w) h_f + (C_s/C_c)A_s + (C_f/C_c)[b + 2h] \rightarrow \text{Minimum} \quad (26)$$

Subjected to the design constraints (equations, Eq.(7) through Eq.(25)).

3.5 Formulation of minimum weight design problem of reinforced concrete T-beams

Find the design variables b, b_w, h, d, h_f, A_s, and α that minimize total weight per unit length of reinforced concrete T-beam such that:

$$W = (b_w h + (b - b_w)h_f)\rho \longrightarrow \text{Minimum} \quad (27)$$

Subjected to the design constraints (equations, Eq.(7) through Eq.(25)).

3.6 Solution methodology

The objective function Eq. (26), the objective function Eq. (27) and the constraints equations, Eq.(7) through Eq.(25), together form a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are essentially due to the expressions of the cross sectional area, bending moment capacity and other constraints equations. Both the objective function and the constraint functions are nonlinear in terms of the design variables. In order to solve this nonlinear optimization problem, the generalized reduced gradient (GRG) algorithm is used. The Generalized Reduced Gradient method is applied as it has the following advantages: i) The GRG method is widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems. ii) The program can handle up to 200 constraints which is suitable for reinforced ordinary and HSC beams design optimization problems.

4. Numerical results and discussion

4.1 Design example A for reinforced HSC T-beams

The numerical example A corresponds to a high strength concrete T-beam belonging to a bridge deck, simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

The corresponding pre-assigned parameters are defined as follows:

L = 25m; M_{Ed}= 1.35M_G+ 1.5M_Q=9MNm; V_{Ed} = 1.35V_G+ 1.5V_Q= 3.1MN
w=0.60MN/ml; δ_{lim}=L/250=0.100m.

Input data for HSC characteristics:
C70/85;f_{ck} = 70MPa; γ_c =1.5;f_{cd}= 46.67MPa; ρ= 0.025MN/m^3; E_{cm}=40743MPa
λ=0.75; η=0.90; ε_{cu3}(‰)=2.7; ε_{c3}(‰)=2.4; h_{fmin}=0.10m; f_{ctm}= 4.6MPa
μ_{limit}= 0.329; α_{limit}=0.554 for S500 and C70/85

Input data for steel characteristics:
S500; f_{yk}=500MPa; γ_s=1.15; $f_{yd}=f_{yk}/\gamma_s$= 435MPa; n=15
S400; f_{yk}=400MPa; γ_s =1.15; $f_{yd}=f_{yk}/\gamma_s$= 348MPa; f_{yd}/f_{cd}=9.32 for classes (S500, C70/85)
f_{yd}/f_{cd} =7.46 for classes (S400, C70/85); μ_{limit}= 0.352; α_{limit}=0.6081 for S400 and C70/85
E_s=2x10^{5}MPa; ρ_{min}=0.26f_{ctm}/f_{yk}=0.002392; ρ_{max} = 4%.

Input data for units costs ratios of construction materials:
C_s/C_c = 40 for HSC concrete
C_f/C_c = 0.01for wood formwork
C_f/C_c = 0.10for metal formwork
C_f/C_c = 0.00in the case of the cost of the formwork is negligible

4.2 Comparison between the minimum cost design and the minimum weight design of HSC T-beams

The vector of design variables including the geometric dimensions of the T-beam cross section and the area of tension reinforcement as obtained from the standard design approach solution and the optimal cost design solution using the proposed approach, are shown in Table 2.

The optimal solutions using the minimum cost design and the minimum weight design are shown in Table 2 below.

Table 2: Comparison of the optimal solutions with minimum weight and minimum cost design for HSC.

Vector solution	Classical solution	Optimal solution with minimum cost (S500, C70/85) C_s/C_c=40, C_f/C_c=0.01 wood formwork	Optimal solution With minimum weight
b(m)	1.20	0.86	0.52
b_w(m)	0.40	0.28	0.28
h(m)	1.40	1.58	1.56
d(m)	1.26	1.42	1.40
h_f(m)	0.15	0.11	0.10
A_S(m^2)	185x10^{-4}	161x10^{-4}	181 x10^{-4}
α	0.554	0.342	0.554
Gain		22%	47%

It can be seen from the Table 2 that the gain and the optimum values for minimum cost design and for minimum weight design are different.
From the above results, it is clearly shown that a significant cost saving of the order of 47% can be obtained using the proposed minimum weight design formulation and 22% through the use of minimum cost design approach.

4.3 Parametric study

In this section, the optimal solution is obtained according to practical consideration (i) the total depth is imposed $h = h_{imposed}$, (ii) the effective width of compressive flange is imposed $b = b_{imposed}$, (iii) the reinforcing steel is imposed $A_s = A_{simposed}$ and (4i) the flange depth is imposed $h_f = h_{fimposed}$.
The gain depends on the type of formwork used. We distinguish the wood formwork C_f/C_c= 0.01 and the steel formwork C_f/C_c= 0.10.
Further practical requirements can also be implemented, such as aesthetic, architectural and limited authorized template. The optimal solutions obtained using the particular conditions imposed are shown in Table 3 below.

Table 3: Variation of relative gain with particular conditions imposed such as the HSC T-beam dimensions and reinforcing steel.

Optimal solution with	Gain
Classes(S500, C70/85); C_s/C_c=40; C_f/C_c=0.01wood formwork	22%
Classes(S500, C70/85); C_s/C_c=40; C_f/C_c=0.10 steel formwork	19%
Classes(S500,C70/85) and C_f/C_c=0 the cost of the formwork is negligible	23%
Classes(S400,C70/85); C_s/C_c=40; C_f/C_c=0.01wood formwork	08%
Imposed height h=1.70m; S500 and C70/85	21%
Imposed width b=1.00m; S500 and C70/85	22%
Imposed reinforcement A_s≤0.0150m^2; S500 and C70/85	22%
Imposed flange depth h_f =0.10m; S500 andC70/85	22%

From the above results, it is clearly seen that a significant cost saving between 08% and 23%can be obtained by using this parametric study.

4.4 Sensitivity analysis

The relative gains can be determined for various values of the unit cost ratios:
C_s/C_c=10; 20; 30; 40; 50; 60; 70; 80; 90; 100 for a given unit cost ratio C_f/C_c=0.01
The corresponding results are reported in Table 4 and represented in Figure 2.

Table 4: Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio C_f/C_c=0.01

(S500; C70/85) C_f/C_c=0.01	Gain
10	33%
20	27%
30	24%
40	22%
50	22%
60	22%
70	23%
80	24%
90	26%
100	27%

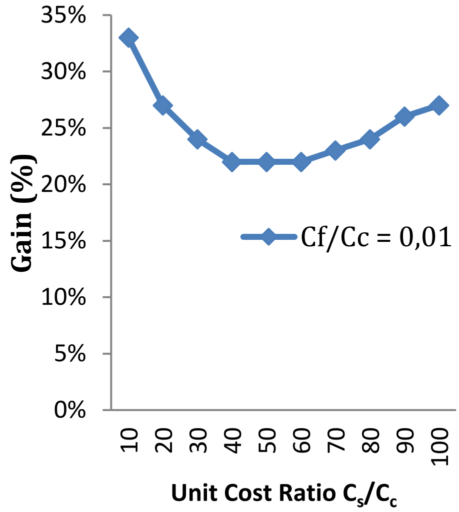

Figure 2 Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio C_f/C_c =0.01

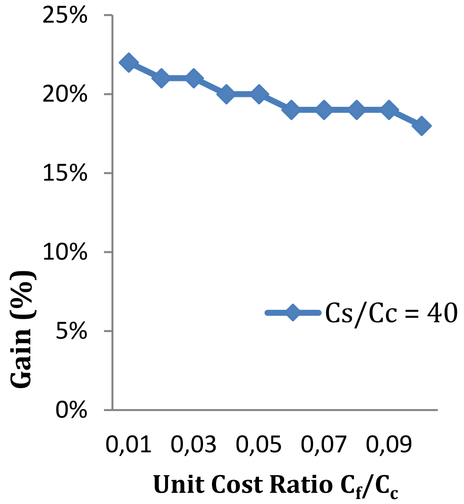

Figure 3 Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for a given cost ratio C_s/C_c =40

It can be observed from the Table 4 and the Figure 2, that the relative gain decreases for increasing values of the unit cost ratio C_s/C_c, stabilizes around an average value for $40 \leq C_s/C_c \leq 60$ and then increases significantly beyond this average value for a given cost ratio C_f/C_c=0.01.

The relative gains can be determined for various values of the unit cost ratios:
C_f/C_c=0.01; 0.02; 0.03; 0.04; 0.05; 0.06; 0.07; 0.08; 0.08; 0.09; 0.10 for a given unit cost ratio C_s/C_c=40.
The corresponding results are reported in Table 5 and presented in Figure 3.

Table 5: Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for a given cost ratio C_s/C_c=40

(S500; C70/85) C_s/C_c=40 C_f/C_c	Gain
0.01	22%
0.02	21%
0.03	21%
0.04	20%
0.05	20%
0.06	19%
0.07	19%
0.08	19%
0.09	19%
0.10	18%

From the above Table 5 and the Figure 3, the gain decreases monotonically with the increasing of the unit cost ratio $C_f/C_c$for a given cost ratio C_s/C_c=40.

4.5 Design example B for reinforced ordinary concrete T-beams

The numerical example B corresponds to a concrete T-beam belonging to a pedestrian deck, simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

The pre-assigned parameters are defined as follows:
L = 20m; M_{Ed}= 5MNm; V_{Ed} = 1.1MN;
w=0.043MN/ml; δ_{lim}=L/250=0.080m

Input data for ordinary concrete characteristics:
C20/25; f_{ck} = 20MPa; γ_c =1.5;f_{cd}= 11.33MPa; ρ= 0.025MN/m^3; E_{cm}=30000MPa
λ=0.80; η =1.00; ε_{cu3}(‰)=2; ε_{c3}(‰)=3.5;
h_{fmin}=0.15m; f_{ctm}= 2.20MPa; n=15
μ_{limit}= 0.372; α_{limit}=0.6167 for S500 and C20/25
μ_{limit}= 0.392; α_{limit}=0.6680 for S400 and C20/25

Input data for steel characteristics:
S400; f_{yk} = 400MPa; γ_s =1.15; f_{yd}= f_{yk}/γ_s =348MPa
E_s =2x10^{5}MPa; ρ_{min} = 0.26f_{ctm}/f_{yk}=0.00143;
ρ_{max} = 4%
f_{yd}/f_{cd}=30.71 for classes (S400, C20/25)
f_{yd}/f_{cd}=38.39 for classes (S500, C20/25)

Input data for units costs ratios of construction materials:
$C_s / C_c = 30$ for ordinary concrete
$C_f / C_c = 0.10$ for metal formwork
$C_f / C_c = 0.01$ for wood formwork

4.6 Comparison between the minimum cost design and the minimum weight design of ordinary concrete T-beams

The optimal solutions using the minimum weight design and the minimum cost design are shown in Table 6 below.

Table 6: Comparison of the optimal solutions with minimum weight and minimum cost design

Vector solution	Classical solution C20/25 & S400	Optimal solution with minimum weight C20/25 & S400	Optimal Solution With minimum cost C20/25 & S400
b(m)	1.20	1.30	1.25
b_w(m)	0.40	0.28	0.29
h(m)	1.60	1.57	1.60
d(m)	1.44	1.41	1.44
h_f(m)	0.14	0.17	0.16
$A_S(m^2)$	$125x10^{-4}$	$123x10^{-4}$	$122x10^{-4}$
α	0.668	0.668	0.668
C	1.171		1.0281
Gain			14%

It can be seen from the Table 6 that the gain and the optimum values for minimum weight design and for minimum cost design are different.
From the above results, it is clearly shown that a significant cost saving of the order of 23% can be obtained using the proposed minimum weight design formulation and 14% through the use of minimum cost design approach.

4.7 Parametric study

In this section, the optimal solution is obtained through the consideration (i) one of the dimensions of HSC T-section is imposed h = 1.50m, (ii) imposed reinforcing steel $A_s=120x10^{-4}m^2$, (iii) imposed web width $b_W=0.30m$ and (4i) imposed relative depth of compressive concrete zone α=0.6000.

Further practical requirements can also be implemented, such as aesthetic, architectural and limited authorized template.

The optimal solutions obtained using the particular conditions imposed are shown in Table 7 below.

Table 7: Variation of relative gain with particular conditions imposed such as the T-beam dimensions, reinforcing steel and weight.

Optimal solution with	Gain
$f_{yd}/f_{cd}=30.71$; $C_s/C_c=30$; $C_f/C_c=0.01$ wood formwork, C20/25 & S400	14%
$f_{yd}/f_{cd}=38.39$; $C_s/C_c=30$; $C_f/C_c=0.01$wood formwork, C20/25 & S500	09%
$f_{yd}/f_{cd}=30.71$; $C_s/C_c=30$; $C_f/C_c=0.00$; C20/25 & S400	15%
Imposed web with b_w= 0.30m; $f_{yd}/f_{cd}=30.71$; $C_s/C_c=30$; $C_f/C_c=0.01$; C20/25 & S400	13%
Imposed reinforcement$A_s \leq 0.0120m^2$; $f_{yd}/f_{cd}=30.71$; $C_s/C_c=30$; $C_f/C_c=0.01$; C20/25 & S400	14%
Imposed height h=1.50m; $f_{yd}/f_{cd}=30.71$; $C_s/C_c=30$; $C_f/C_c=0.01$; C20/25 & S400	11%
Imposed relative depth α=0.600; $f_{yd}/f_{cd}=30.71$; $C_s/C_c=30$; $C_f/C_c=0.01$; C20/25 & S400	14%

From the above results, it is clearly seen that a significant cost saving between 09% and 15%can be obtained by using this parametric study.

4.8 Sensitivity analysis

The relative gains can be determined for various values of the unit cost ratios:
C_s/C_c=10; 20; 30; 40; 50; 60; 70; 80; 90; 100 for a given unit cost ratio $C_f/C_c=0.01$
The corresponding results are reported in Table 8 and presented graphically in Figure 4.

Table 8: Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio $C_f/C_c=0.01$

(S400; C20/25) $C_f/C_c=0.01$	Gain
10	18%
20	16%
30	14%
40	13%
50	12%
60	12%
70	12%
80	11%
90	11%
100	11%

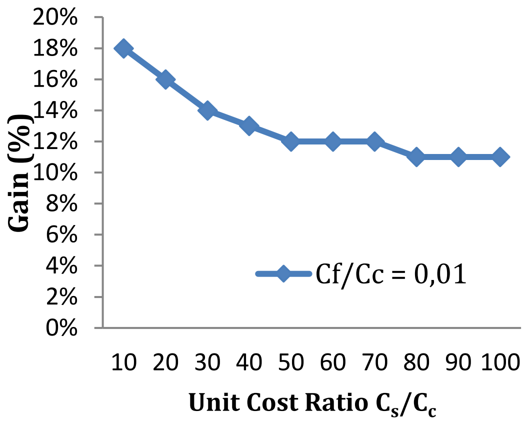

Figure 4 Variation of relative gain in percentage (%) versus unit cost ratio C_s/C_c for a given cost ratio C_f/C_c =0.01

It can be observed from Table 8 and Figure 4, that the relative gain decreases for increasing values of the unit cost ratio C_s/C_c for a given value of C_f/C_c=0.01.

The relative gains can be determined for various values of the unit cost ratios:
C_f/C_c=0.01; 0.02; 0.03; 0.04; 0.05; 0.06; 0.07; 0.08; 0.08; 0.09; 0.10 for a given unit cost ratio C_s/C_c=30
The corresponding results are reported in Table 9 and illustrated graphically in Figure 5.

Table 9: Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for C_s/C_c=30

(S400; C20/25) C_s/C_c=30 C_f/C_c	Gain
0,01	14%
0,02	14%
0,03	13%
0,04	13%
0,05	13%
0,06	12%
0,07	12%
0,08	12%
0,09	12%
0,1	12%

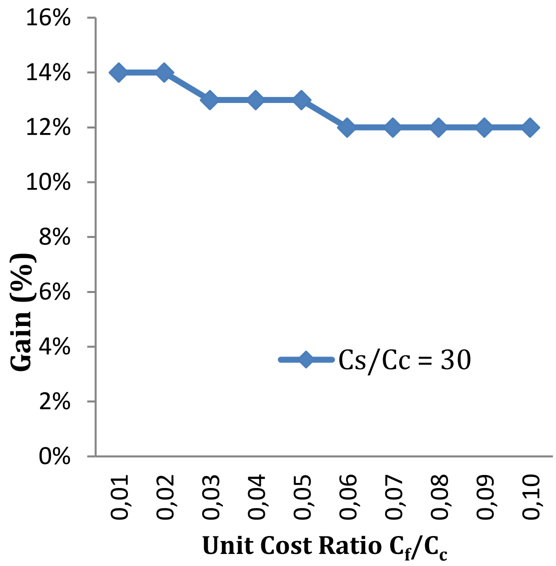

Figure 5 Variation of relative gain in percentage (%) versus unit cost ratio C_f/C_c for a given cost ratio C_s/C_c =30

From the Table 9 and Figure 5, the gain decreases monotonically with the increasing of the unit cost ratio C_f/C_c

For a given value of C_s/C_c=30.

5. Conclusions

The following important conclusions are drawn on the basis of this study:

- The problem formulation of the optimal cost design of reinforced concrete T-beams can be cast into a nonlinear programming problem, the numerical solution is efficiently determined using the GRG (Generalized Reduced Gradient) method in a space of only a few variables representing the concrete cross section dimensions.
- The space of feasible design solutions and the optimal solutions can be obtained from a reduced number of independent design variables.
- The optimal values of the design variables are only affected by the

relative cost values of the objective function and not by the absolute cost values.

- The optimal solutions are found to be insensitive to changes in the shear constraint. Shear constraint is not usually critical in the optimal design of reinforced concrete T-beams under bending and thus can be excluded from the problem formulation.
- The observations of the optimal solutions results reveal that the use of the optimization based on the optimum cost design concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of reinforced concrete T-beams.
- The objective function and the constraints considered in this paper are illustrative in nature. This approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design. More sophisticated objectives and considerations can be readily accommodated by suitable modifications of the optimal cost design model.
- In this work, we have included the additional cost of formwork which makes a significant contribution to the total costs. This integration is important for an economical approach to design and manufacture.
- The suggested methodology for optimum cost design is effective and more economical comparing to the classical methods. The results of the analysis show that the optimization process presented herein is effective and its application appears feasible.
- Comparison of optimal solutions for minimum cost and minimum weight shows that the construction cost affects significantly the optimal sizes. Not only do we use the mass, but the cost as objective function as well which contains the material and construction provisions costs. The difference is caused by the construction details costs.

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Appendix

List of symbols

The following symbols are used in this paper:

C20/25 Class of ordinary concrete
C70/85 Class of HSC
S400 Grade of steel
S500 Grade of steel
f_{ck} Characteristic compressive cylinder strength of ordinary or HSC at 28 days
f_{ctm} Tensile strength of concrete
f_{cd} Design value of concrete compressive strength
γ_c Partial safety factor for concrete
η Design strength factor
λ Compressive zone depth factor
ε_{c3} Strain at the maximum stress for the rectangular stress distribution compressive concrete
ε_{cu3} Ultimate strain for the rectangular stress distribution compressive concrete design stress-strain relation f_{yk} Characteristic elastic limit for steel reinforcement
γ_s Partial safety factor for steel.
f_{yd} Design yield strength of steel reinforcement
ε_{yd} Elastic limit strain
E_s Young's elastic modulus of steel
E_{cm} Modulus of elasticity of concrete
ρ_{min} Minimum steel percentage
ρ_{max} Maximum steel percentage
α_{limit} Limit value of relative depth of compressive concrete zone
μ_{limit} Limit value of reduced moment
L Beam span
w The mid-span deflection of simply supported beam under distribution load w (dead load+ live load) for reinforced concrete T-beam
V_G Maximum design shears under dead loads
V_Q Maximum design shears under live loads
$V_{Rd,max}$ Maximum resistant shear force
V_{Ed} Ultimate shear force
$M_{Rd, max}$ Maximum resisting moment
M_{Ed} Ultimate bending moment
M_G Maximum design moments under dead loads
M_Q Maximum design moments under live loads
F_s Resultant tensile internal force for steel

F_c Resultant compressive internal force for HSC
n Ratio of the modulus of elasticity of steel to that of concrete
b Effective width of compressive flange
b_w Web width
h Total depth
h_f Flange depth
d Effective depth
d_s Effective cover of reinforcement.
As Area of reinforcing steel
h_{fmin} Minimum depth of flange
δ_{lim} Limit deflection
θ Angle between concrete compression struts and the main chord.
ν_1 A non dimensional coefficient; $\nu_1=0.60(1-f_{ck}/250)$
z Lever arm, z=0.9d
ρ Density of the reinforced concrete T-beams
W Unit weight per unit length of the reinforced concrete T- beams.
C_0/L Total cost per unit length of T-beam
C_s Unit cost of reinforcing steel
C_c Unit cost of concrete
C_f Unit cost of formwork